Computational Continua

Jacob Fish and Sergey Kuznetsov

Multiscale Science and Engineering Center

Rensselaer Polytechnic Institute

Troy, NY 12180

Abstract

We present a new continuum description hereafter referred to as computational continua. The label "computational" is conceived due to both theoretical and computational considerations. From theoretical point of view, not only that the computational continua is endowed with detailed information about the fine-scale features, but more importantly, it introduces no scale separation, makes no assumption about infinitesimality of the fine-scale structure and requires no *a priori* knowledge of the nonlocal kernel function. From computational point of view, the computational continua does not require higher order continuity; it introduces no new degrees-of-freedom; and it is free of higher order boundary conditions. The computational continuum description features two new concepts: the nonlocal quadrature scheme and the coarse-scale stress function. The nonlocal quadrature scheme injects the finite size effect into the classical two-scale integrals used in homogenization. The nonlocal quadrature scheme is implicitly related to the nonlocal kernel function, but is defined at the nonlocal quadrature points only. The quadrature points, which define positions of so-called computational unit cell centroids, and the value of the kernel function there, are chosen to ensure consistency with the governing equations on the fine scale. The coarse-scale stress function, which replaces the classical notion of coarse-scale stress being the average of fine-scale stresses, allows restating the governing equations of continua in terms of coarse-scale fields only. Perhaps the most intriguing finding of this work is that the coarse-scale continuum description that is consistent with an underlying fine-scale description cannot be separated from the discretization scheme employed as is the usual case in the homogenization and generalized continua theories. As a prelude to introducing the computational continua framework, we show the relation between the generalized continua and higher order mathematical homogenization theory and point out to their limitations. This serves as motivation to the main part of the presentation, which is the computational continua formulation.